I. 1) $E^*_{in} = \frac{\rho_0}{2\varepsilon_0}\vec{r}$ Ⓐ ~~1~~ 0,5 et $E^*_{ex} = \frac{\rho_0}{2\varepsilon_0}\frac{a^2}{r^2}\vec{r}$ Ⓑ $= 0,5$

$V_m(\vec r) = \frac{1}{\rho_0}\vec{P}.\vec{E^*}(\vec r)$ et $\vec{E}_m = -\overrightarrow{grad}V_m = -\frac{\vec P}{\rho_0}.\overrightarrow{grad}\left[\vec{E^*}(\vec r)\right]$

d'où $\vec{E}_{m,in} = -\frac{P}{2\varepsilon_0}\frac{\partial}{\partial x}(x.\vec{e_x}+y.\vec{e_y}) = -\frac{\vec P}{2\varepsilon_0}$ Ⓒ $= 1$

~~0,5~~ $\vec{E}_{m,ex} = -\frac{P}{2\varepsilon_0}a^2\frac{\partial}{\partial x}\left(\frac{x\,\vec{e_x}+y\,\vec{e_y}}{x^2+y^2}\right) = -\frac{P}{2\varepsilon_0}\frac{a^2}{r^4}\left[(r^2-2x^2)\vec{e_x}-2xy.\vec{e_y}\right]$

$= \frac{P}{2\varepsilon_0}\frac{a^2}{r^2}(\vec{e_x}.\cos 2\varphi + \vec{e_y}.\sin 2\varphi)$ Ⓓ $= 1$

2) $\vec{E}_{int} = \vec{0} = \vec{E_a} + \vec{E}_{m,in} = \vec{E_a} - \frac{P}{2\varepsilon_0}\vec{e_x} \Rightarrow \vec{P} = 2\varepsilon_0\vec{E_a}$ Ⓔ $= 1$

$\sigma = \vec{P}.\vec{n}_{ex} = P\vec{e_x}.\vec{e_r} = P\cos\varphi = 2\varepsilon_0 E_a\cos\varphi$ Ⓕ $= 0,5$

$\vec{p} = \vec{P}.\mathcal{V} = 2\varepsilon_0\pi a^2 l\,\vec{E_a}$ Ⓖ $= 0,5$

3) $\vec{E}_{ex} = \vec{E}_{m,ex} + \vec{E_a} = \underbrace{E_a\left[\left(1+\frac{a^2}{r^2}\cos 2\varphi\right)\vec{e_x} + \frac{a^2}{r^2}\sin 2\varphi\,\vec{e_y}\right]}_{Ⓗ = 0,5}$

$\vec{D}_{in} = \varepsilon_0\vec{E}_{in} + \vec{P} = \vec{P}$ Ⓘ $= 0,5$

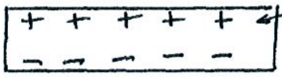
$\vec{D}_{ex} = \varepsilon_0\vec{E}_{ex} = \frac{P}{2}\left[\left(1+\frac{a^2}{r^2}\cos 2\varphi\right)\vec{e_x} + \frac{a^2}{r^2}\sin 2\varphi.\vec{e_y}\right]$ Ⓙ $= 0,5$

II. 1) $\vec{E}\begin{cases}\vec{E}_{in} = \frac{\varepsilon_0}{\varepsilon}\vec{E_a}\\ \vec{E}_{ex} = \vec{E_a}\end{cases}$ Ⓐ ~~1~~ $\quad \vec{D}\begin{cases}\vec{D}_{in} = \varepsilon_0\vec{E_a}\\ \vec{D}_{ex} = \varepsilon_0\vec{E_a}\end{cases}$ Ⓑ $= 0,5$ $\quad \vec{P}\begin{cases}\vec{P}_{in} = \vec{D}_{in} - \varepsilon_0\vec{E}_{in} = \varepsilon_0\vec{E_a}\left(1-\frac{\varepsilon_0}{\varepsilon}\right)\\ \vec{P}_{ex} = \vec{0}\end{cases}$ Ⓒ $= 1$

On a bien $\vec{E}_{in} = \vec{E}_{m,in} + \vec{E_a} = -\frac{\vec P}{\varepsilon_0} + \vec{E_a} = -\left(\frac{\varepsilon}{\varepsilon_0}-1\right)\vec{E}_{in} + \vec{E_a}$

~~5~~ pts

Distribution des charges :



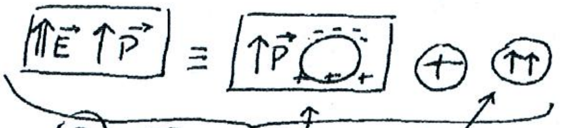
$\left.\begin{array}{l}\sigma_{in} = \vec{P}.\vec{n}\\ \vec{n} = \pm\vec{e_x}\end{array}\right\}$ Ⓓ $= 0,5$ $\quad \sigma_{in} = \pm P$; continuité de D_n.

2) $\vec{D}_1 = \varepsilon_1\vec{E}_1$ et $\vec{D}_2 = \varepsilon_2\vec{E}_2$; $\vec{D}_1 \overset{Ⓔ = 0,5}{=} \vec{D}_2 = \vec{D}_{ex} \overset{Ⓕ = 0,5}{=} -\sigma_{ex}.\vec{e_x}$

$U = e_1E_1 + e_2E_2 = e_1\frac{D_1}{\varepsilon_1} + e_2\frac{D_2}{\varepsilon_2} = \left(\frac{e_1}{\varepsilon_1}+\frac{e_2}{\varepsilon_2}\right)D = \left(\frac{e_1}{\varepsilon_1}+\frac{e_2}{\varepsilon_2}\right)\left(\frac{Q}{ab}\right)$ Ⓖ $= 0,5$

$C = \frac{a.b}{\frac{e_1}{\varepsilon_1}+\frac{e_2}{\varepsilon_2}} = \frac{Q}{U} \Rightarrow \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow$ 2 condensateurs en série. Ⓗ $= 0,5$

III . 1) $\vec{E}_\ell = \vec{E} + \frac{\vec{P}}{3\varepsilon_0}$ (A) = 1 ,



(B) $\vec{E}_\ell = \vec{E}_\ell(1) + \vec{E}_\ell(2)$ = 1 , $\vec{E}_\ell(2) = \vec{0}$ pour amorphes (C) = 0,5

2) Dipôle macroscopique moyen $\langle p \rangle = \varepsilon_0 \cdot \alpha_{or} \cdot E_\ell$

où α_{or} = polarisation d'orientation = $\frac{p_0^2}{3\varepsilon_0 kT}$ (D) = 1

p_0 = dipôle permanent moléculaire.

3) $\vec{P} = \varepsilon_0 \underbrace{(\varepsilon_r - 1)}_{\chi} \vec{E} + N \cdot \varepsilon_0 \alpha_{or} \cdot \underbrace{\left(\vec{E} + \frac{\vec{P}}{3\varepsilon_0}\right)}_{\vec{E}_\ell}$. (E) = 1

4) $\vec{P}\left[1 - \frac{N\alpha_{or}}{3}\right] = \varepsilon_0 \vec{E}\left[\varepsilon_r - 1 + N\alpha_{or}\right]$ (F) = 0,5 $\Rightarrow \vec{P} = \frac{\varepsilon_0 \vec{E}(\varepsilon_r - 1 + N\alpha_{or})}{1 - \frac{N\alpha_{or}}{3}}$ (G) = 0,5

9 pts

5) $\varepsilon_r)_{total} = 1 + \chi_{total} = \frac{1 - \frac{N\alpha_{or}}{3} + \varepsilon_r - 1 + N\alpha_{or}}{1 - \frac{N}{3}\alpha_{or}}$ (H) = 1

$\varepsilon_r)_{total} = \frac{\varepsilon_r + \frac{2}{3}N\alpha_{or}}{1 - \frac{N}{3}\alpha_{or}}$ (I) = 0,5 $\simeq \left(\varepsilon_r + \frac{2}{3}N\alpha_{or}\right)\left(1 + \frac{N\alpha_{or}}{3}\right) \simeq \varepsilon_r + (N\alpha_{or}) \times \ldots \left(\frac{\varepsilon_r}{3} + \frac{2}{3}\right)$ (J) = 1

6) Continuité de $\vec{D}$: $\varepsilon_0 \vec{E}_{ex} = \varepsilon_0 (\varepsilon_r)_{total} \cdot \vec{E}_{in}$

$\Rightarrow \vec{E}_{in} = \frac{1}{(\varepsilon_r)_{total}} \cdot \vec{E}_{ex}$ (K) = 1